Oiras' *chou* Dimensional Expansion

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Summary

・The dimensional expansion of Menelaus' theorem, Manelaus' Theorem is proved.

・The inverse of Manelaus' Theorem cannot be valid.

・Ceva's Theorem in 3-D is proved.

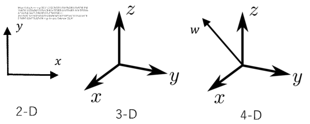
・We suggest some ways of solving the kissing number problem.

Motive

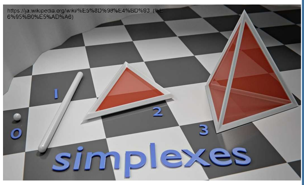
One of Sora's dreams is to make Doraemon. It is necessary to know other dimensional worlds in order to make it. So we decided to elucidate them with mathematics.

Definition

n-D
A world with n primary independent coordinate axes.



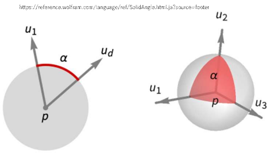
n-simplex
A generalization of the notation of a triangle or a tetrahedron to arbitrary dimensions.



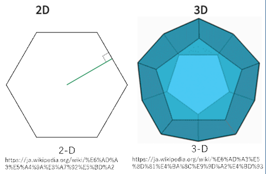
n-face A generalization of the notation of a vertex or a edge or a face

n-hyper volume A generalization of the notation of length or square or volume. $s(C_0C_1C_2\ldots C_n)$ stands for the hyper volume of simplex $C_0C_1C_2\ldots C_n$.

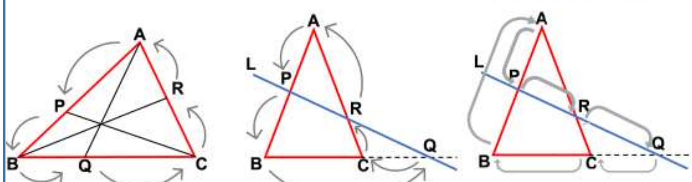
n-unit ball
A generalization of the notation of a unit circle or a unit ball.



polytope
A generalization of the notation of a polygon or a polyhedron.



Manelaus' Theorem



$$\frac{AP}{PB}\cdot\frac{BQ}{QC}\cdot\frac{CR}{RA}=1$$

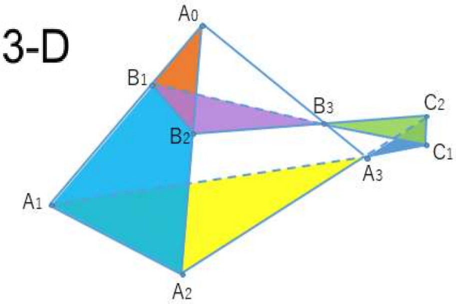
https://univ-juken.com/menerausu

$$\frac{QC}{CB}\cdot\frac{BA}{AP}\cdot\frac{PR}{RQ}=1$$

Cut a m-simplex $A_0A_1A_2\cdots A_m$ by (m-1)-hyperplane $B_1B_2\cdots B_m$ and make $C_1C_2\cdots C_{m-1}$ for their intersection.

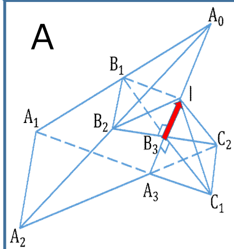
$$\frac{s(A_mC_1C_2\cdots C_{m-1})}{s(A_1A_2A_3\cdots A_m)}\cdot\frac{s(A_0A_1A_2\cdots A_{m-1})}{s(A_0B_1B_2\cdots B_{m-1})}\cdot\frac{s(B_1B_2B_3\cdots B_m)}{s(B_mC_1C_2\cdots C_{m-1})}=1$$

※In 3-D



No Inverse of Manelaus' Theorem

A

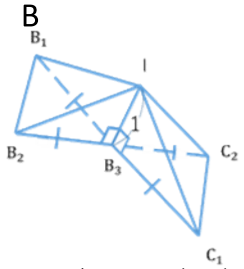


From Manelaus' Theorem,

$$\frac{s(A_3C_1C_2)}{s(A_1A_2A_3)}\cdot\frac{s(A_0A_1A_2)}{s(A_0B_1B_2)}\cdot\frac{s(B_1B_2B_3)}{s(B_3C_1C_2)}=1$$

…①

B



From Picture B,

$$\frac{s(B_1B_2B_3)}{s(B_3C_1C_2)}=\frac{s(B_1B_2I)}{s(IC_1C_2)}$$

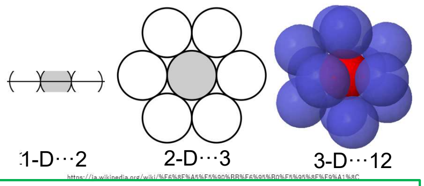
…②

・ $\frac{s(A_3C_1C_2)}{s(A_1A_2A_3)}\frac{s(A_0A_1A_2)}{s(A_0B_1B_2)}\frac{s(B_1B_2I)}{s(IC_1C_2)}=1$ ∵①, ②

This is Counterexample,
thus the Inverse of Manelaus' Theorem.

Kissing Number Problem

How many m-unit balls can you arrange so that they touch one m-unit ball without overlapping?



https://ja.wikipedia.org/wiki/%E6%8E%A5%E5%90%BB%E6%95%B0%E5%95%8F%E9%A1%8C

1.Oiras' chou Polytope theorem･･･

Euler's polyhedron formula can be expanded in n-D.

About a m-polytope, we named it A. We also named the number of n-face f(m_A,n).

$$X(A)=\sum_{n=0}^{m-1}(-1)^n f(m\_A,n)=1$$

The centers of the unit balls
→The most dense regular polytope

$$n\left\{\frac{{}_mC_1}{a_{m-1}-1}-\frac{{}_mC_2}{2}+\frac{{}_mC_3}{2}-\ldots+(-1)^{m-1}\frac{{}_mC_m}{2}+(-1)^m\right\}=1$$

(m=3,4)

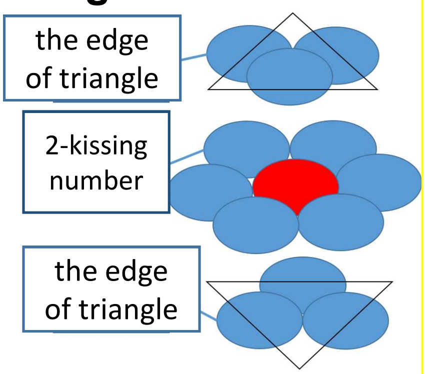
2. m-dimensional Electronic Capacity Theory

Add up the numbers of s, p, d orbital and subtract the number of symmetrical d orbit like d_{xy}, d_{yz}, d_{zx}, and multiply by the number of spin. Define f_m(n) as electronic capacity of m-dimensional atom's *n*th shell.

a_m=(m-1){f_m(1)+f_m(2)+f_m(3)－${}_mC_2$}
(3≦m≦5)

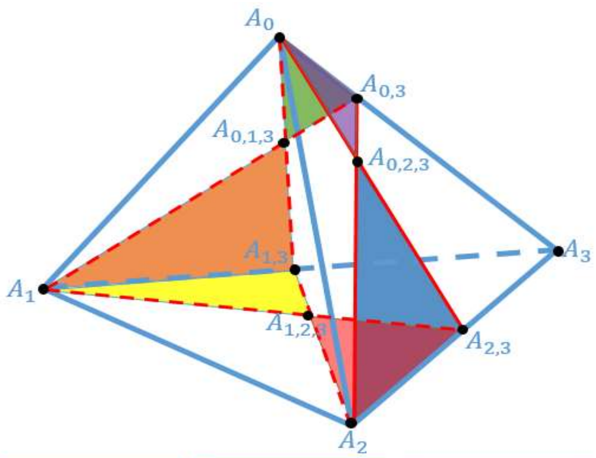
3.Attention to the number of edges

We define m-kissing number as a_m, and the number of a m-simplex' edges is ${}_{m+1}C_2$.



$a_{m+1}=a_m$**+2${}_{m+1}C_2$** (2≦m≦5)

Ceva's Theorem in 3-D



$$\frac{s(A_{1,2,3}A_{2,3}A_2)}{s(A_{1,2,3}A_{1,3}A_1)}\cdot\frac{s(A_{0,1,3}A_{1,3}A_1)}{s(A_{0,1,3}A_{0,3}A_0)}\cdot\frac{s(A_{0,2,3}A_{0,3}A_0)}{s(A_{0,2,3}A_{2,3}A_2)}=1$$

Future Prospect

We would like to prove…
・Ceva's Theorem in n-D
・theorems with a line and a polygon (except for a triangle) in n-D

References

・Sora Tazaki etc.(Our Research)
"'Oiras' *chou* Polytope Theorem' Deciding Euler Characteristic of Polytopes "
(超多面体のオイラー標数を定める『オイラらの*chou*多面体定理』)
・平面図形の定理の空間への拡張
・Sora Tazaki (Our leader)
「電子収容数次元理論～電子軌道とスピンの次元一般化～」
・平面図形の定理の空間への拡張
https://www.nagano-c.ed.jp/seiho/intro/risuka/kadaikenq/paper/2021/7_heimen.pdf
(2022.9.15)
・チェバ・メネラウスの定理に関する教材開発：n角形への拡張
https://shizuoka.repo.nii.ac.jp/?action=pages_view_main&active_action=repository_view_main_item_detail&item_id=11389&item_no=1&page_id=13&block_id=21
(2022.9.15)